| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 | 6 | 6 | 9 | / | 0 | 1 | Signature | |

Paper Reference(s)

**6669/01**

# Edexcel GCE

## Further Pure Mathematics FP3

## Advanced/Advanced Subsidiary

Tuesday 23 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae (Orange) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



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**Turn over**

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Leave blank

**1.** Solve the equation

$$7\operatorname{sech} x - \tanh x = 5$$

Give your answers in the form $\ln a$ where $a$ is a rational number.

**(5)**

Leave blank

**2.**

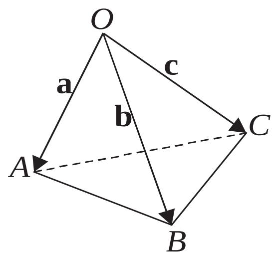


**Figure 1**

The points $A$, $B$ and $C$ have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, relative to a fixed origin $O$, as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$,

**(3)**

(b) $\mathbf{a}.(\mathbf{b} \times \mathbf{c})$,

**(2)**

(c) the area of triangle $OBC$,

**(2)**

(d) the volume of the tetrahedron $OABC$.

**(1)**

Leave blank

**Question 2 continued**

Q2

**(Total 8 marks)**

Leave blank

**3.**

$$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(a) Show that 7 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

**(5)**

(b) Find an eigenvector corresponding to the eigenvalue 7.

**(4)**

Leave blank

**Question 3 continued**

Leave blank

**4.** Given that $y = \text{arsinh}\,(\sqrt{x}),\quad x > 0,$

(a) find $\dfrac{dy}{dx}$, giving your answer as a simplified fraction.

**(3)**

(b) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\,dx\,,$$

giving your answer in the form $\ln\left(\dfrac{a+b\sqrt{5}}{2}\right)$, where $a$ and $b$ are integers.

**(6)**

Leave blank

**5.**

$$I_n = \int_0^5 \frac{x^n}{\sqrt{(25-x^2)}}\,\mathrm{d}x\,, \qquad n \geqslant 0$$

(a) Find an expression for $\int \frac{x}{\sqrt{(25-x^2)}}\mathrm{d}x$, $\quad 0 \leqslant x \leqslant 5$.

**(2)**

(b) Using your answer to part (a), or otherwise, show that

$$I_n = \frac{25(n-1)}{n} I_{n-2} \qquad n \geqslant 2$$

**(5)**

(c) Find $I_4$ in the form $k\pi$, where $k$ is a fraction.

**(4)**

Leave blank

**Question 5 continued**

Leave blank

**6.** The hyperbola $H$ has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, where $a$ and $b$ are constants.

The line $L$ has equation $y = mx + c$, where $m$ and $c$ are constants.

(a) Given that $L$ and $H$ meet, show that the $x$-coordinates of the points of intersection are the roots of the equation

$$(a^2m^2 - b^2)x^2 + 2a^2mcx + a^2(c^2 + b^2) = 0$$

**(2)**

Hence, given that $L$ is a tangent to $H$,

(b) show that $a^2m^2 = b^2 + c^2$.

**(2)**

The hyperbola $H'$ has equation $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$.

(c) Find the equations of the tangents to $H'$ which pass through the point (1, 4).

**(7)**

Leave blank

**Question 6 continued**

Leave blank

**7.** The lines $l_1$ and $l_2$ have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines $l_1$ and $l_2$ intersect, find

(a) the value of $\alpha$,

**(4)**

(b) an equation for the plane containing the lines $l_1$ and $l_2$, giving your answer in the form $ax + by + cz + d = 0$, where $a$, $b$, $c$ and $d$ are constants.

**(4)**

For other values of $\alpha$, the lines $l_1$ and $l_2$ do not intersect and are skew lines.

Given that $\alpha = 2$,

(c) find the shortest distance between the lines $l_1$ and $l_2$.

**(3)**

Leave blank

**Question 7 continued**

Leave blank

**8.** A curve, which is part of an ellipse, has parametric equations

$$x = 3\cos\theta, \quad y = 5\sin\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

The curve is rotated through $2\pi$ radians about the $x$-axis.

(a) Show that the area of the surface generated is given by the integral

$$k\pi\int_0^{\alpha} \sqrt{(16c^2 + 9)}\,\mathrm{d}c, \quad \text{where } c = \cos\theta,$$

and where $k$ and $\alpha$ are constants to be found.

**(6)**

(b) Using the substitution $c = \frac{3}{4}\sinh u$, or otherwise, evaluate the integral, showing all of your working and giving the final answer to 3 significant figures.

**(5)**


M 3 5 1 4 5 A 0 2 4 2 8

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**Question 8 continued**

Q8

**(Total 11 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**